5/15/17"The most important thing you can give someone is a chance."-C. Berman

HW: "Intro to Probability" homework section #1-9
Test 2 on Thursday 5/18

AIM: What is Probability?

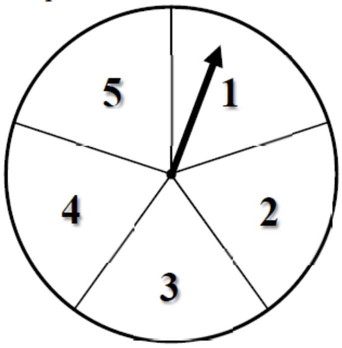
Warm Up:

Write "Probability" as the Topic on the MIT

**BASIC PROBABILITY TERMINOLOGY**

1. **Experiment:** Some process that occurs with well defined outcomes.
2. **Outcome:** A result from a single **trial** of the experiment.
3. **Event:** A collection of one or more outcomes.
4. **Sample Space:** A collection of all of the outcomes of an experiment.

***Exercise* #1:** An experiment is run whereby a spinner is spun around a circle with 5 equal sectors that have been marked off as shown.




(a) What is the **experiment**?

Spinning the spinner

(b) Give one outcome of the experiment.

Spinning a 3

(c) What is the probability of spinning the spinner and landing on an odd number? What is the event here? What outcomes fall into the event?

Spinning an odd

1, 3, 5

$P(\text{odd}) = \frac{3}{5}$ or .6 or 60%

The answer from (c) helps us to define the basic formula that dictates all probability calculations:

**THE BASIC DEFINITION OF PROBABILITY**

The probability of an event $E$ occurring is given by the ratio: $P(E) = \frac{n(E)}{n(S)}$, where:

$P(E) = \frac{\text{want}}{\text{total possible}}$

$n(E)$ is the number of outcomes that fall into the event $E$

$n(S)$ is the number of outcomes that fall into the sample space

***Exercise* #2:** Given the above definition, between what two numbers must ALL probabilities lie? Explain.

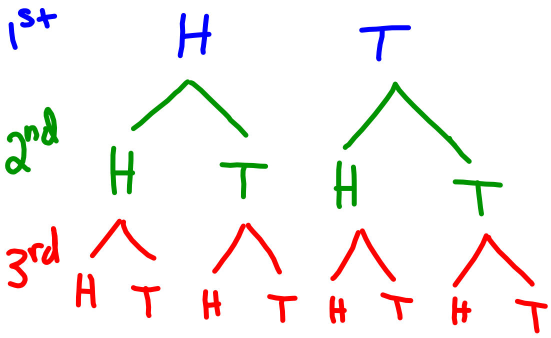
$0 \le \text{Probability} \le 1$

$[0, 1]$

When we deal with **theoretical probability** we don't actually have to run the experiment to determine the probability of an event. We simply have to know the number of outcomes in the sample space and the number of outcomes that fall into our event. Let's take a look at a slightly more challenging scenario.

***Exercise* #3:** A fair coin is flipped three times and the result is noted each time. The sample space consists of **ordered triples** such as $(H, H, T)$, which would represent a head on the first toss, a head on the second toss, and a tail on the third toss.

(a) Draw a **tree diagram** to show all of the different outcomes in the sample space.




(b) List all of the outcomes as ordered triples. How many of them are there?

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

c) (i) P(all heads) $= \frac{1}{8}$

(ii) P(Exactly 2 heads) $= \frac{3}{8}$

(iii) P(All H or All T) $= \frac{2}{8} = \frac{1}{4}$

Sometimes we have to quantify chance by using observations that have been made in the real-world. In this case we talk about **empirical probability**. The fundamental equation for probability still stands.

***Exercise* #4:** A survey was done by a marketing company to determine which of three sodas was preferred by people in a blind taste test. The results are shown below.

(a) Find the empirical probability that a person selected at random from this group would prefer soda B. Express your answer as a fraction and as a decimal accurate to two decimal places (the standard).

| Soda | Number who Preferred |
|---|---|
| A | 18 |
| B | 24 |
| C | 11 |
| Total | 53 |

P(soda B) $= \frac{24}{53}$ or .45

(b) Find the empirical probability that a person selected at random from this group would *not* prefer soda A. Again, express your answer as a fraction and as a decimal accurate to two decimal places.

P(not Soda A) $= \frac{24+11}{53} = \frac{35}{53}$ or .66

Since the basic calculation within probability involves counting the number of **outcomes** that fit into a particular **event**, it makes sense to have a tool to visualize and keep track of all of the outcomes in a **sample space**. We will do this by using sets. Recall the basic definition of a set:

**SET DEFINITION**

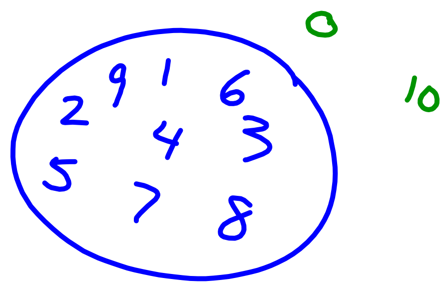
A **set** is simply a collection of things (numbers, objects, etcetera) that satisfy a well-defined criteria. The things that are contained in the set are called the **elements** of the set

***Exercise* #1:** The set A is defined as the collection of all integers that are greater than 0 and less than 10.

(a) Write out set A in **roster form**.

$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(b) Show set A in **Venn Diagram** form. This will be a very simple Venn Diagram.




(c) A **subset** is any set whose elements are all contained within another set. Give two possible rules that could define subsets of A and then write the sets as B and C in roster form. Do sets B and C have any elements in common?

Set B's Definition: All integers greater than 3 $B = \{4, 5, 6, 7, 8, 9\}$

Set C's Definition: All even integers $C = \{2, 4, 6, 8\}$

Let's get back to a bit of probability.

***Exercise* #2:** Consider an experiment where we first toss a coin and note the outcome and then roll a six-sided die and note the outcome.

(a) Write a set of ordered pairs, such as $(H, 4)$, that represents all outcomes for this experiment. Recall that this is called the **sample space**. We will generally call this set S.

(H,6) (H,2) (T,4)
(H,5) (H,1) (T,3)
(H,4) (T,6) (T,2) = S
(H,3) (T,5) (T,1)

(b) Write a set of ordered pairs that represents the event of getting a tail and an even number. Call this set A.

A={(T,2) (T,4) (T,6)}

(c) The complement of a set A will be all of the events in the sample space S that do not fall into set A. Write out the complement of set A. We'll call this set B.

B={(H,1)(H,2)(H,3)(H,4)
(H,5)(H,6)(T,1)(T,3)(T,5)}

(d) Find $P(A)$ and $P(B)$.

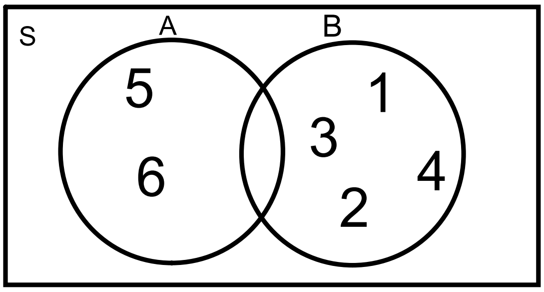
P(A)=3/12

P(B)=9/12

A set and its complement are important when we look at probability because all outcomes either fall into an event or into its complement, but not both.

***Exercise* #3:** Consider rolling a single six-sided die and recording the result. Let set A be the event of rolling a number greater than 4 and let set B be the complement of set A.

(a) Draw a Venn Diagram that illustrates the sample space, S, and sets A and B.




(b) Find $P(A)$ and $P(B)$.

P(A)=2/6

P(B)=4/6

(c) What is true of the sum $P(A)+P(B)$?

2/6 + 4/6 = 1

(d) Prove that the sum of the probability of an event with the probability of its complement will always be 1.

We use the relationship developed in (d) all the time without even thinking about it. Try the following.

***Exercise* #4:** Answer each of the following problems by using the relationship developed in Exercise #3(d).

(a) If the probability I will draw a red marble from a bag is $\frac{3}{17}$, what is the probability that I won't draw a red marble from a bag?

1-(3/17)= 14/17

(b) If the probability that it will rain tomorrow is 20%, what is the probability that it won't rain tomorrow?

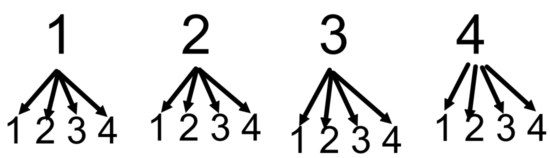
1-.2= .8

80%

In theoretical probability calculations, the sets that make up the sample spaces can get difficult to write out. It is good to remember things like tree diagrams to help.

***Exercise* #5:** Two four-sided die are rolled and the number on each is noted.

(a) Draw a tree diagram that represents all outcomes in the sample space. How many are there?




16 outcomes

(b) What is the probability that you don't get two of the same number?

P(not same)=12/16